

# Steep inflation: Ending braneworld inflation by gravitational particle production

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We propose a scenario for inflation based upon the braneworld picture, in which high-energy corrections to the Friedmann equation permit inflation to take place with potentials ordinarily too steep to sustain it. Inflation ends when the braneworld corrections begin to lose their dominance. Reheating may naturally be brought about via gravitational particle production, rather than the usual inflaton decay mechanism; the reheat temperature may be low enough to satisfy the gravitino bound and the Universe becomes radiation dominated early enough for nucleosynthesis. We illustrate the idea by considering steep exponential potentials, and show they can give satisfactory density perturbations (both amplitude and slope) and reheat successfully. The scalar field may survive to the present epoch without violating observational bounds, and could be invoked in the quintessential inflation scenario of Peebles and Vilenkin.

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## I. INTRODUCTION

The realization [1] that we may live on a so-called brane embedded in a higher-dimensional universe has significant implications for cosmology. One such implication is that the Friedmann equation is modified at very high energies [2,3], acquiring a term quadratic in the density. Such a term generally makes it easier to obtain inflation in the early Universe by contributing extra friction to the scalar field equation of motion [4,5].

In this paper we propose to capitalize on this feature by considering potentials which are normally too steep to support inflation. As we will see, this allows a particularly natural implementation of an unusual model for reheating at the end of inflation. In the conventional inflationary scenario, reheating is achieved by decay of the inflaton field into normal matter, either via parametric resonance or by single particle decays. However, there are at least two other mechanisms which have been proposed in the literature. One is reheating via the production of primordial black holes [6], whose subsequent Hawking evaporation produces the conventional matter—we do not discuss this possibility further. The second, which we exploit here, is that particles arise from gravitational particle production, with the inflaton energy density subsequently redshifting sufficiently quickly that this produced radiation comes to dominate. This scenario was first discussed by Ford [7] (see also Ref. [8]) and refined by Spokoiny [9]; recent applications include baryogenesis [10], scaling solutions in the present Universe [11], and the “quintessential inflation” scenario of Peebles and Vilenkin [12,13].

In conventional inflationary models, the gravitational production scenario is difficult to arrange. In order to redshift the inflaton density more quickly than that of the produced particles one needs a sharp feature in the potential, so that it

becomes steep enough that the inflaton becomes kinetic energy dominated, leading to  $\rho_\phi \propto 1/a^6$ . The gravitational particle production scenario arises much more naturally in the context of inflation on the brane. When one considers a potential which would ordinarily be too steep to support inflation, the transition to a kinetic energy dominated regime occurs naturally once the energy density falls sufficiently for the quadratic term to begin to lose its dominance. Thus there is no need for a feature in the inflaton potential.

Although our scenario could be implemented for a wide range of inflaton potentials, for simplicity we focus our discussion on the well-studied exponential potentials [14]

$$V(\phi) = V_0 \exp\left(-\sqrt{8\pi}\alpha \frac{\phi}{M_4}\right), \quad (1)$$

where  $M_4$  is the four-dimensional Planck mass and  $\alpha$  is a constant. In the standard cosmology, the potential is shallow enough to support inflation only if  $\alpha^2 < 2$ .

We consider the five-dimensional brane scenario, in which the Friedmann equation is modified from its usual form, becoming [2,3]

$$H^2 = \frac{8\pi}{3M_4^2} \rho \left[ 1 + \frac{\rho}{2\lambda} \right] + \frac{\Lambda_4}{3} + \frac{\mathcal{E}}{a^4}, \quad (2)$$

where  $\Lambda_4$  is the four-dimensional cosmological constant and the final term represents the influence of bulk gravitons on the brane. The brane tension  $\lambda$  relates the four- and five-dimensional Planck masses via

$$M_4 = \sqrt{\frac{3}{4\pi}} \left( \frac{M_5^2}{\sqrt{\lambda}} \right) M_5, \quad (3)$$

and is constrained by the requirement of successful nucleosynthesis as  $\lambda > (1 \text{ MeV})^4$ . We assume that the four-dimensional cosmological constant is set to zero by some (as yet undiscovered) mechanism, and once inflation begins the final term will rapidly become unimportant, leaving us with

$$H^2 = \frac{8\pi}{3M_4^2} \rho \left[ 1 + \frac{\rho}{2\lambda} \right]. \quad (4)$$

We assume that the scalar field is confined to the brane, so that its field equation has the standard form

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}. \quad (5)$$

## II. CONSTRAINTS

We analyze our model following closely the formalism of Maartens *et al.* [4]. As we will see later, suitable values of  $\alpha$  will be greater than about five; such potentials would not give inflation in the conventional cosmology.

### A. Inflationary dynamics

Following Maartens *et al.* [4], we define a slow-roll parameter  $\epsilon$ , generalizing the usual ones [15], by

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{M_4^2}{4\pi} \left( \frac{V'}{V} \right)^2 \frac{1 + V/\lambda}{(2 + V/\lambda)^2}, \quad (6)$$

where prime indicates a  $\phi$  derivative and the slow-roll approximation has been employed. Inflation takes place whenever  $\epsilon < 1$ . For our model we can always take  $V \gg \lambda$  during inflation, so

$$\epsilon \approx \frac{2\alpha^2\lambda}{V}. \quad (7)$$

The end of inflation will take place when  $\epsilon = 1$ , giving

$$V_{\text{end}} \approx 2\alpha^2\lambda. \quad (8)$$

Notice that even at the end of inflation, typically the term quadratic in the density still dominates the linear term by a significant factor.

The amount of inflation that takes place is given by the number of  $e$ -foldings [4]

$$N \approx -\frac{8\pi}{M_4^2} \int_{\phi}^{\phi_{\text{end}}} \frac{V}{V'} \left( 1 + \frac{V}{2\lambda} \right) d\phi. \quad (9)$$

As  $V \gg \lambda$  during inflation, this simply becomes

$$N \approx \frac{1}{2\lambda\alpha^2} (V_N - V_{\text{end}}), \quad (10)$$

and so the potential  $V_N$  a given number of  $e$ -foldings from the end of inflation is given by

$$V_N = V_{\text{end}}(N + 1). \quad (11)$$

### B. Perturbations and the COBE normalization

The amplitude of density perturbations is given by [4]

$$A_S^2 \approx \frac{512\pi}{75M_4^6} \frac{V^3}{V'^2} \left( 1 + \frac{V}{2\lambda} \right)^3 \approx \frac{8}{75} \frac{V^4}{M_4^4\alpha^2\lambda^3}. \quad (12)$$

The observed value from COBE is  $A_S = 2 \times 10^{-5}$  [16], which we apply 50  $e$ -foldings from the end of inflation; using Eqs. (8) and (11) we obtain the simple result

$$V_{\text{end}}^{1/4} = \frac{9 \times 10^{-5}}{\alpha} M_4 = \frac{1 \times 10^{15} \text{ GeV}}{\alpha}. \quad (13)$$

Equivalently, this fixes the brane tension as

$$\lambda \approx \frac{4 \times 10^{-17}}{\alpha^6} M_4^4 = \left( \frac{10^{15} \text{ GeV}}{\alpha^{3/2}} \right)^4, \quad (14)$$

or the five-dimensional Planck mass as

$$M_5 = \frac{2.3 \times 10^{-3}}{\alpha} M_4 = \frac{3 \times 10^{16} \text{ GeV}}{\alpha}. \quad (15)$$

For consistency it should be verified that enough inflation can indeed occur. The assumption that the scalar field is confined to the brane becomes unreliable if  $V > M_5^4$ . Imposing this condition along with the Cosmic Background Explorer (COBE) normalization leads to the simple result

$$N_{\text{max}} = 4 \times 10^5, \quad (16)$$

independent of both  $\alpha$  and  $\lambda$ . This large dimensionless number has its origin in the small dimensionless number corresponding to the observed temperature anisotropy. It may be that more inflation is possible before the scalar field becomes confined to the brane, but the observable last 50 or so  $e$ -foldings will be in that regime.

The spectral index of the density perturbation spectrum can also be found following Ref. [4], leading to

$$n = 1 - \frac{4}{N+1}. \quad (17)$$

This result is independent of both  $\alpha$  and the brane tension, and with the usual assumption of 50  $e$ -foldings gives  $n = 0.92$ . This is compatible with current observations (e.g., see Ref. [17]), but can be strongly tested in the future.

In the high-energy limit of the braneworld scenario the contribution of gravitational waves relative to density perturbations is suppressed, and the relative amplitude is given by [18]

$$\frac{A_T^2}{A_S^2} \approx \frac{3M_4^2}{16\pi} \left( \frac{V'}{V} \right)^2 \frac{2\lambda}{V} \approx 0.03, \quad (18)$$

which again is independent of all model parameters. This is a significant level of production, not far from current upper limits; in terms of the commonly used quantity  $r \simeq 4\pi A_1^2/A_8^2$  we have  $r \simeq 0.4$ , and it is believed that the Planck satellite will eventually measure  $r$  with an error bar of around 0.05.

### C. Reheating by gravitational particle production

At the end of inflation, the scalar field density is given by  $V_{\text{end}}$ . Some of this energy needs to be converted to conventional matter to restore the hot big bang cosmology. Usually this is brought about by decay of the inflaton field, though this could only proceed to completion if the potential is modified to introduce a minimum.

Instead, we take advantage of the possibility of reheating by gravitational particle production [7,9,11], where the required particles are produced quantum mechanically from the time-varying gravitational field. Standard calculations [7,9] give the density of particles produced at the end of inflation as

$$\rho_R \sim 0.01 g_{\text{prod}} H_{\text{end}}^4 \simeq 0.2 g_{\text{prod}} \frac{V_{\text{end}}^4}{\lambda^2 M_4^4}, \quad (19)$$

where  $g_{\text{prod}}$  is the number of fields experiencing particle production, likely to be between 10 and 100. This result depends only on the fields' equations of motion and the expansion rate, and so remains valid in the braneworld scenario. The relative densities at the end of inflation are

$$\frac{\rho_R}{\rho_\phi} = 0.2 g_{\text{prod}} \frac{V_{\text{end}}^3}{\lambda^2 M_4^4} = 5 g_{\text{prod}} \times 10^{-17}, \quad (20)$$

where the last equality uses the COBE normalization, Eqs. (13) and (14). The numerical value of the radiation density, given the COBE normalization, is

$$\rho_R \simeq g_{\text{prod}} \left( \frac{10^{11} \text{ GeV}}{\alpha} \right)^4, \quad (21)$$

which if immediately thermalized would give a temperature

$$T_{\text{end}} \simeq \frac{10^{11} \text{ GeV}}{\alpha} \left( \frac{g_{\text{prod}}}{g_*} \right)^{1/4}, \quad (22)$$

where  $g_*$  is the total number of species in the thermal bath (which may be somewhat higher than  $g_{\text{prod}}$ ). This temperature is around that at which thermal production of gravitinos via two-particle collisions becomes problematic [19], and only more detailed calculations (including the amount of cooling during the final thermalization) can clarify whether the gravitino limit can be satisfied. It also needs to be determined whether or not non-thermal gravitino production might be significant (see, e.g., Ref. [20]). See Ref. [13] for other model-building difficulties that may arise in constructing a working scenario.

Although the ratio of radiation to scalar field densities at the end of inflation is small, it is possible for the radiation to

become dominant. Once the high-energy correction to the Friedmann equation becomes unimportant, the potential is so steep that the evolution becomes completely kinetic-energy dominated, with  $\rho_\phi \propto 1/a^6$ . We first assume this behavior sets in as soon as inflation ends, and examine the opposite case in the next paragraph. When the kinetic energy is completely dominant, we have

$$\frac{\rho_R}{\rho_\phi} \propto a^2. \quad (23)$$

From Eq. (20), we can estimate that the radiation comes to dominate after the Universe has expanded by a factor around  $10^7$  to  $10^8$  after inflation, at which stage the temperature, which obeys  $T \propto 1/a$ , is

$$T \simeq \frac{10^3 \text{ GeV}}{\alpha}. \quad (24)$$

We see that radiation domination sets in comfortably before nucleosynthesis.

This needs some correction to allow for the period between the end of inflation and the onset of validity of the usual Friedmann equation at  $\rho \simeq 2\lambda$ , which will cause a delay before Eq. (23) applies. As inflation has ended, we know that  $\rho_\phi$  must fall off at least as quickly as  $1/a$ .<sup>1</sup> This means a worst-case scenario where the scale factor has increased by an extra factor of  $(V_{\text{end}}/2\lambda)^{5/6} = \alpha^{5/3}$  by the time the usual Friedmann equation sets in, as compared to the argument of the previous paragraph. The radiation temperature at that time is therefore smaller by that factor, and so extra expansion by a factor  $\alpha^{10/3}$  will be required to bring on radiation domination. For typical parameters this may reduce the temperature at the onset of radiation domination by a factor between a hundred and a thousand as compared to Eq. (24), which is still early enough for successful nucleosynthesis. In any event, this worst-case scenario will not be attained in practice, leading to a higher temperature at the onset of radiation domination.

Notice that while in this cosmology the formal epoch of radiation domination does not begin until quite a low temperature [Eq. (24), corrected as described in the previous paragraph], the radiation originally was created at a much higher temperature [Eq. (22)]. It may well be therefore that processes such as baryogenesis might occur within the radiation fluid at an epoch before it comes to dominate the density of the Universe—see Ref. [10].

### D. Scaling in the present Universe

The period of kinetic domination will not last indefinitely; it is well known that the late-time behavior for steep exponential potentials is a scaling solution where the scalar field

<sup>1</sup>In the standard cosmology this would be  $1/a^2$ , coming from the inflationary condition  $p < -\rho/3$ , but in the high-energy limit of brane cosmology the inflationary condition becomes  $p \lesssim -2\rho/3$  [4].

exhibits the same redshift dependence as the dominant fluid in the Universe [21,22]. The scaling density, assuming spatial flatness, is

$$\Omega_\phi = \frac{3(w+1)}{\alpha^2}, \quad (25)$$

where the dominant fluid has equation of state  $p = w\rho$ . If the scaling solution is already attained by nucleosynthesis, the field acts as an extra relativistic species and will adversely affect the standard picture unless  $\alpha^2 \gtrsim 20$  [11,22]. If the scaling solution is not attained by nucleosynthesis (as may well be the case as the kinetic energy has a tendency to overshoot the scaling solution by several orders of magnitude before approaching it from below [11]) this limit can be evaded, but a similar limit can be derived from the effect of the scalar field on density perturbations at the present epoch [11].

Because the scalar field survives to the present, a suitable feature in the potential near its present value can give rise to quintessence behavior; this is a version of the quintessential inflation scenario of Peebles and Vilenkin [12] in which the same scalar field gives both early Universe inflation and the present observed acceleration. More attractively, one could use a different form of the potential, such as an inverse power law, which allows a natural transition to quintessence at late times and which merits further investigation. Our model allows such a scenario with a much simpler choice of potential than in Einstein gravity models due to our exploitation of the braneworld effects.

### III. CONCLUSIONS

We have presented a non-standard model of inflation, made possible by braneworld-motivated corrections to the Friedmann equation at high energies. The novel ingredients are that inflation becomes possible for a class of potentials

ordinarily too steep to sustain accelerated expansion, with the end of inflation brought on by the braneworld corrections losing their dominance. While standard reheating could be considered, this scenario allows a particularly natural implementation of reheating via gravitational particle production, and we have confirmed that all existing constraints can be satisfied.

For the particular implementation of this idea using steep exponential potentials, a distinctive prediction is that the spectral index of density perturbations is given by  $n \approx 0.92$ , independent of all parameters (except the number of  $e$ -foldings to the end of inflation). This prediction, along with the large predicted amplitude of gravitational waves, is readily testable with upcoming experiments.

Although we have not attempted to place this scenario within the context of a realistic particle physics model, it should be emphasized that current M-theory models have a number of likely candidate scalar fields with steep exponential potentials living on the brane. These include the dilaton in five-dimensional heterotic M theory [23], the dilaton present in self-tuning mechanisms to cancel the cosmological constant on the brane [24], and combinations of the dilaton and moduli fields that arise in heterotic M theory when studying the cosmological stabilization of these fields [25]. The important point is that all of these models would be affected by the presence of the corrections to the Friedmann equation, and intriguingly this could open up the possibility of inflation occurring before the moduli fields become stabilized.

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